ELLIPTIC INTERFERENCE WITH REFLECTING GRATING.

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1. *First Method.*—There are two or three typical cases in the use of reflecting gratings for the production of interferences in the spectrum, each of which shows peculiarly interesting features. The first of these is given in Fig. 1 and corresponds closely to the method described for transmission gratings in a preceding paper. If L is the source of light and M a glass plate grating, it was shown that
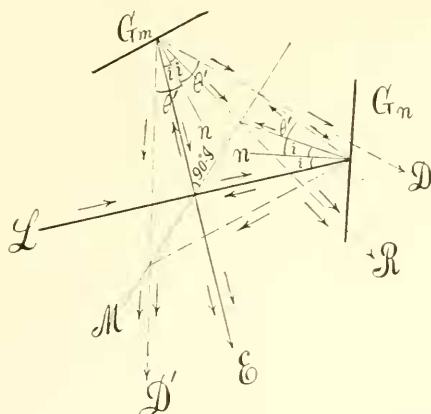

Fig. 1. Diagram, showing positions of mirror, M, and grating, G.

plane mirrors in the positions G_m and G_n, each reflecting a spectrum from M, produce elliptical interference whenever the rays returned after passing M by transmission and reflection, respectively, are made to overlap in the spectrum, under suitable conditions.

The present method is the converse of this, since the gratings and the opaque mirrors change places. Parallel rays from L strike the plate of glass M and the component rays reach identical reflecting

gratings G_m and G_n, placed symmetrically with respect to M at an angle i to the E and L directions. The undeviated rays pass off eccentrically at R and are not seen in the telescope at E. They may, however, be seen in an auxiliary telescope pointed in the line R and they then facilitate the adjustments. Rays diffracted at the angle $2i$, however, are respectively transmitted and reflected by M and interfere in the telescope in the line E. Similarly rays diffracted at an angle $\theta' > i$ interfere in the line D.

To make the adjustment it is sufficient to bring the Fraunhofer lines in the two spectra seen at E into complete coincidence, horizontally and vertically. Coincidence of slit images at R (at least vertically) aids in the same result. It is also necessary that the rulings on G_m and G_n and the slit should be parallel, or that the images of slit and spectra shall lie between the same horizontals. One of the gratings, G_n, may now be moved parallel to itself by the micrometer screw until the elliptic interferences appear. If the plate M is not half silvered there are three groups of these as described in the preceding paper. Each group passes from the initial degree of extreme fineness, through maximum size, to a final degree, for a play of the screw of about 1 mm. There is the usual radial motion of the fringes, together with the drift through the spectrum as a whole. To bring out the set of solitary ellipses, the silvered surface of M should be towards the light and remote from the eye. As a rule the adjustment is difficult, as an extra condition is imposed in the parallelism of the slit and the rulings of the gratings. The ellipses are liable to be coarse with their axes oblique, clearer in some parts of the spectrum than in others, unless means are provided for placing the rulings accurately parallel. Even when well adjusted they are rather polygonal than rounded in their contours. They are about as strong with non-silvered glass M as with half-silvered glass; but in view of the multiple spectra the adjustment is much more difficult in the former case.

It has been suggested that the white slit images must appear eccentrically in the direction R. Hence if a special telescope is directed in this line, the final adjustment is reached on coincidence of the proper slit images, provided the rulings of the gratings and the slit are parallel.

For $\theta' > i$ the second series of interference spectra occurring at D, eccentrically, are broader, but only on perfect adjustment do they occur simultaneously with the other set.   In fact, since for the preceding case $i = \theta$, or

$$2 \sin i = \lambda/D$$

and in the present case,

$$\sin \theta' - \sin i = \lambda/D,$$

therefore

$$\sin \theta' = 3 \sin i = 3 \sin \theta.$$

There is also an available set in the second order to the left of E. In the gratings used above D lies in front of G_n. being nearer the E than the L direction.

2. *Inversion of the Method.*—The occurrence of the undeviated ray R suggests another method.   For if the white ray R is *reversed,* i. e., comes from an eccentric collimator, slit images will be seen in telescopes at L and E, whereas overlapping spectra will appear in the direction D' eccentrically and in the lines R and R'.   One of the latter may be lost in the collimator.   The former occurs for the same angle θ' so that

$$\sin \theta' = 3 \sin i.$$

Moreover, if $I = 45°$ is the angle of incidence of L upon M when sodium light is taken, so that $\theta' = 26° 14'$, $i = 8° 28'$, the R, D, D' rays make angles $2i$, $\theta' + i$, $\theta' - i$, respectively, with the E direction ; or the sum of the angles at D and D' with the E line is $2\theta'$, their difference $2i$, and the rays D, R, D' intersect at a common centre on G_m.   Hence if we place the plane of G_m at the centre of the spherometer and arrange M and G_n eccentrically, the angles may be measured as before.

3. *Resolution of the Slit Image.*—If the sharp white images of the slit in a Michelson apparatus for the case in which the incident light consists of parallel white rays from a collimator, be accurately superimposed and the opaque mirrors be set at the proper distances from the semi-transparent mirror by the micrometer, the slit image may itself be viewed through a grating and will then show elliptic interferences in all the spectra.   The apparatus is here eccentric,

while the grating (either transmitting or reflecting) must be at the center of the spectrometer, if angles are to be measured. The same is true for any of the other superimposed white slit images in the above or the earlier experiments and may even be repeated with successive transmitting gratings. It is interesting to note that the position of the center of ellipses is at the same wave length in all the spectra though the form of ellipses may differ enormously. The same phenomenon may thus be seen by a number of observers at the same time, each looking through his own telescope.
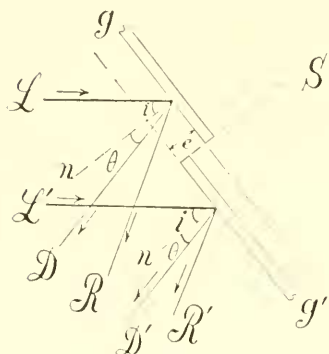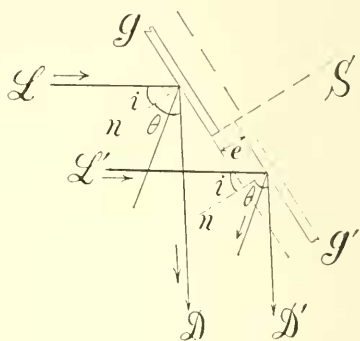

FIG. 2.   FIG. 3.
Diagrams showing position of gratings, g, g'.

4. *Third Method. Parallel Gratings.*—In this case the two halves of the grating are treated displaced parallel to themselves, from their original coplanar position in the grating, from which they are cut. Their mounting is thus something like the case of the two black plates of Fresnel's mirror apparatus, one of the plates being adapted for displacement parallel to itself.

In Fig. 2 g and g' show the two halves of the grating cut along the plane S, normal to the plates and parallel to the rulings. The half g' is provided with a micrometer screw, so that it may be successively moved from the position g' in Fig. 2 to the position g' in Fig. 3, through all intermediate positions, while the half g remains stationary. Each of the halves g and g' is controlled by three adjustment screws (horizontal and vertical axes of rotation), to secure complete parallelism of the faces of the grating. Each, moreover,

may be rotated around a horizontal axis to place the lines parallel to the slit of the collimator. The duplex grating is mounted on a spectrometer as is usual for reflection. Finally each half may be raised and lowered and moved horizontally to and fro, parallel to itself, so that the half gratings when coplanar may approximately reproduce the original grating.

After each of the spectra are clear as to Fraunhofer lines, the interferences here in question are produced by bringing these lines (the D lines for instance) into perfect coincidence, horizontally and vertically. Under these circumstances if the distance apart, c, is suitably chosen, the interference fringes will appear throughout the spectrum. These consist of an approximately equidistant series of lines parallel to the slit, *i. e.*, vertical lines, which are finer, *cæt. par.*, as the breadth of the crack at S between the gratings is larger. They may be increased from the extreme fineness as they enter the range of visibility to a maximum coarseness (in the above experiments) of about three to five minutes per fringe, after which they vanish. They cannot, in practice, be passed through infinite size; neither can they be produced symmetrically on the two sides of the adjustment for infinite size. They cannot in other words be changed from the positive to the negative condition of appearance.

The occurrences are in fact as follows: if as in Fig 2, $i > \theta$, (parallel white rays coming from L and L', R and R' being reflected, D and D' diffracted rays for the normal n), the grating g' must be in advance or forward of g. If now the airspace c is reduced micrometrically, g' retreating, the lines travel in a given direction (from left to right) through the spectrum, while at the same time they grow continually larger until for a minimum value of c still positive, they vanish as a whole. The period of indistinctness before evanescence is not marked.

On the other hand if $\theta' > i$ as in Fig. 3, the grating g' must be to the rear of g and the air space c is throughout negative. If this is now decreased numerically the lines travel through the spectrum in the opposite direction to the preceding case, while at the same time they coarsen until they vanish as a whole as before. The grating g' is still behind g when this occurs.

Finally if for any suitable value of c the grating g' is moved in its own plane without rotation away from g, so as to widen the crack at S between them, the fringes grow continually finer until they pass beyond visibility, and vice versa; *i. c.,* as the crack at S is made smaller the lines continually coarsen.

5. *Nature of the Evanescence.*—The fact that the lines vanish as a whole and almost suddenly after reaching their maximum distance apart is very peculiar, as is also the fact that they cannot be passed through infinite size or appear symmetrically on both sides of this adjustment. To investigate this case I provided both the collimator and the telescope with slits so that the parts of the grating g and g', from which the interfering pencils come, might be investigated.

If a single vertical slit about 1 mm. wide is passed from right to left toward the objective of the telescope, a black line passes
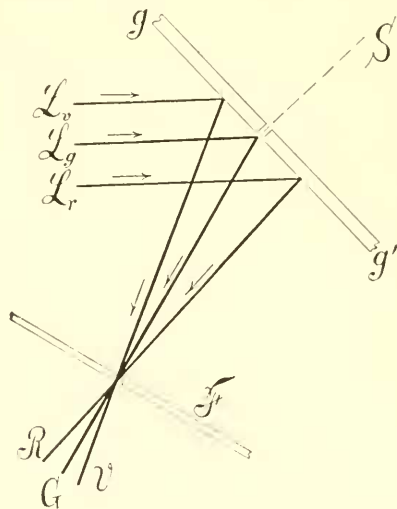

Fig. 4.   Diagram.

across the field of the spectrum, which line is merely the image of the crack at S. In the diagram Fig. 4, the G rays, for instance, come from the edge of both gratings g and g', whereas the R rays and the V rays come from but a single grating. Now when the space e

is diminished, the black band at G gradually vanishes and in its place appear the coarsest fringes producible. When the slit F is removed these coarse fringes disappear. The fringes visible through the slit have however both an inferior and superior limit of angular size. When e is diminished to zero they vanish and when e is sufficiently increased they again vanish, though they now appear when the slit is either removed or widened. From this it follows that the coarsest fringes come from the edges of the crack S of the gratings, and that the remainder of the grating will not produce coarse fringes. By moving the slit the fringes may be made to appear in any other part of the spectrum.

The same fact may be proved by putting the vertical slit F over the lens of the collimator and allowing the white light L to fall on the edges of the grating at S. Coarse fringes limited as to range and size are then seen throughout the spectrum at g.

Whenever the slit or vertical stop is used, the fringes are exceptionally sharp and easily controlled for micrometry. It is not even necessary to adjust the two spectra horizontally with the same care as when no slit is used, but the vertical coincidence of spectrum lines must be sharp. Naturally the use of the slit has one drawback, as the resolving power of the grating is decreased and the spectrum lines are only just visible. The adjustment, however, may be made before the slit is added. A few examples may be given. For a slit 1 mm. wide over the telescope or collimator, only the immediate edges at the crack S, about .5 mm. each in breadth, are active. A narrow range of large fringes are seen in the field easily controlled by the micrometer screw. With a slit 3 mm. in width the lower limit is much increased the upper diminished, to a size of about 3 inches per fringe. In the absence of the slit the field is free from fringes. With a slit 6 mm. wide, the upper limit is again decreased the lower much increased; nevertheless the finest fringes appear only after the slit is removed. Using double slits over the collimator, each 1 mm. wide and 3 mm. apart, fringes of medium size limited at both ends appear; 3 mm. slits 6 mm. apart show only the very fine fringes. but both sizes are still limited. Finally when all but about .5 mm. of the edge of the crack of the grating g' is

screened off, whereas the whole grating g (about one half inch square) is without a screen, all the fringes from the maximum size to complete evanescence beyond the range of visibility are producible. Naturally if the edge of g' is quite dark everything vanishes.

It follows therefore that pairs of corresponding rays are always in question. These corresponding rays are at a definite ND, apart where D is the grating space and N the number of lines per cm. of the grating in question. This distance ND is greater as the fringes are smaller and may be of the order of a cm. when the fringes pass beyond the range of visibility. Again ND is equal to the width of the crack when the largest fringes vanish. Finally when ND is zero, as in the original unbroken grating. the size of the fringes is infinite.

It has been stated that the use of the slit or a laterally limited objective is advantageous because all the lines are much sharper. Inert or harmful illumination is cut off. If the slit is over the objective of the telescope only a small part of the field of view shows the lines; if placed over the objective of the collimator. the fringes are of extreme clearness throughout the spectrum. It may be ultimately of advantage to use the edge near the crack g' only, together with the whole of g. For if a small strip of g' at the crack S is used with the whole of g, the smaller fringes are weakened or wiped out. Thus the inner edge of the nearer grating with successive parts of the further grating is chiefly effective in the production of these interferences.

To bring the two edges quite together was not possible in my work, as they were rough and the apparatus improvised.

7. *Data.*—Some measurements were attempted, with the view only of checking the equations presently to be deduced. The adjustment on an ordinary spectrometer is not firm enough and the fringes being very fine (a few minutes of arc) are difficult to follow unless quite stationary.

Table I., however, gives both the values of de/dn, displacement per fringe, for different angles of incidence i and of diffraction θ, and $d\theta/dn$, the angular deviation per fringe at the D line. In meas-

uring the latter it was necessary to count the fringes between the C and D lines and divide their angular distance apart by these numbers. As c cannot be measured, its successive increments Δc from the first position are given. These are presently to be associated with the corresponding increments of $dn/d\theta$.

TABLE I.

VALUES OF $d\theta/dn$, ETC. $i = 53° 15'$. $D = 200 \times 10^{-6}$ cm.

No. of Fringes.	Observed.					Computed.			Region.
	θ and θ'	$d\theta/dn$	$dn/d\theta$	$\Delta dn/d\theta$	Δe	At θ $\Delta dn/d\theta$	At θ' $\Delta dn/d\theta$	Mean $\Delta dn/d\theta$	
120 75	30° 27' 28° 14' Diff. 2° 13'	1' 17" 1' 46"	3080 } 1950 }	1130	.025	1260	1028	1140	Between C and D lines
90 71 55 36	29° 09' 28° 14' Diff. 55'	46" 1' 0" 1' 32"	4438 } 3438 } 2250	1000 1188	.025 .050 .075	1116	1027	1072	Near C line
24 41	30° 27' 29° 43' Diff. 44'	1' 50" 1' 4"	1875 } 3203 }	1328	.025	1259	1196	1228	Near D line

8. *Equations.*—In Fig. 5, L and L' represent a pair of corresponding white rays, reflected into R and R' and diffracted into D and D' at angles i and θ, respectively. The half gratings g and g' are separated along the crack S, and g' is movable parallel to itself by a micrometer screw normal to g'. Let the normal distance apart of the gratings be c. The incident rays L, L' strike the originally coplanar grating at points N rulings apart, or ND cm. apart, if D is the grating space. In the separated grating let these points be at a distance c apart. Let d be the incident wave front and h the corresponding diffracted wave front and call the angle between c and d, γ.

When there is reinforcement the path difference of the rays L and L' from the incident (d) to the diffracted (h) wave front, may be written

$$n\lambda = b - a,$$

where b and a are the distances of h and d from the points of incidence of L and L' on the grating g and g' respectively. If finally f

is the length of the prolongation of L' between the gratings we may write in succession

(1) $$d = ND \cos i,$$

(2) $$f = c \sec i,$$

(3) $$a = ND \sin i - c \sec i,$$

(4) $$\tan \gamma = a/d,$$

(5) $$C = ND \cos i \sec \gamma,$$
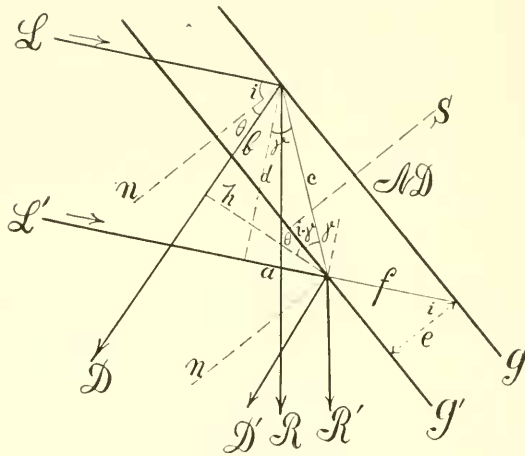
(6) $$b = c \sin (i + \theta - \gamma).$$

Fig. 5.   Diagram.

To these should be added

(7) $$dN/dc = \tan i/D.$$

Hence after removing γ and arranging

$$n\lambda = ND\{\cos i \sin (i + \theta) - \sin i \cos (i + \theta) - \sin i\} + c \sec i(1 + \cos (i + \theta)),$$

which reduces to

$$n\lambda = ND(\sin \theta - \sin i) + c \sec i(1 + \cos (i + \theta)),$$

or since

$$\sin i - \sin \theta = \lambda/D,$$

finally

$$(8) \qquad (n + N)\lambda = c\,\frac{1 + \cos(i + \theta)}{\cos i} = \frac{2c\cos^2(i + \theta)/2}{\cos i}.$$

This must therefore be regarded as the fundamental equation of the phenomenon. Equation (7), however, leads on integration to

$$(9) \qquad\qquad N = c\tan i/D + N_0,$$

where $N_0 D$ is the width of the crack.

If the value of N from (9) is put into (8) together with the equivalent of λ/D, it appears after reduction that

$$(n + N_0)\lambda = c(\cos i + \cos \theta) = 2c\cos\frac{i + \theta}{2}\cos\frac{i - \theta}{2}.$$

The case of $N = 0$, $c > 0$ would correspond to the equation

$$(10) \quad n\lambda = c\left[1 + \cos(i + \theta)\right]\Big/\cos i = 2c\cos^2\frac{i + \theta}{2}\Big/\cos i,$$

which is only a part of the complete equation (8). For $i > \theta$, one active half, kh, is necessarily partly behind the other half, $k'h'$, and therefore not adapted to bring out the phenomenon as explained, unless $c = 0$.

9. *Differential Equations. Displacement per Fringe, dc/dn.*— To test equation (8) or (10) increments must be compared. The latter gives at once since N is constant relative to c like i, θ, and λ,

$$(11) \qquad \frac{dc}{dn} = \frac{\lambda}{\cos i + \cos \theta} = \frac{\lambda}{2\cos\dfrac{i + \theta}{2}\cos\dfrac{i - \theta}{2}}$$

which is the interferometer equation when the fringes pass a given spectrum line, like either D line, which is sharp and stationary in the field. Equations (7) and (11), moreover, give after reduction

$$(12) \qquad\qquad dN/dn = \tan i \tan\frac{i - \theta}{2}.$$

Table I. contains values of dc/dn computed from (11), made under widely different conditions ($i > \theta$, $i < \theta$, first and second order). The agreement is as good as the small fringes and the difficulty of getting the grating normal to the micrometer screw in my impro-

vised apparatus admit.   If this adjustment is not perfect N_0 changes
with c.   From equation (12), moreover,

$$(12') \qquad \frac{dN}{dn} = \frac{dN}{d\theta}\frac{d\theta}{dn} = \frac{dN_0}{d\theta}\frac{d\theta}{dn} = \frac{dN_0}{dn},$$

since N_0 is constant only relative to c when θ varies.

10. *Deviation per Fringe, etc.*, $d\theta/dn$, $d\theta/dc$.—These measure-
ments are still more difficult in the absence of special apparatus,
since c is not determinable and the counting of fine flickering fringes
is unsatisfactory; but the order of results may be corroborated by
observing the number of fringes between two Fraunhofer lines, like
the C, D and other lines used.   Differentiating equations (8) and
(10) for variable n, λ, θ, and N (since $dN/d\theta$ is equal to $dN_0/d\theta$,
equation (12')) and inserting $- D \cos \theta \cdot d\theta/dn = d\lambda/dn$, it follows
after arranging that

$$(13) \qquad \frac{d\theta}{dn} = \frac{\lambda^2}{cD}\frac{1 + dN/dn}{1 + \cos (i + \theta)} = \frac{\lambda^2}{cD}\frac{1}{\cos i(\cos i + \cos \theta)}$$

or

$$\frac{d\theta}{dn} = \frac{\lambda}{c \cos i}\cdot \tan \frac{i - \theta}{2}.$$

Combining this with (11)

$$(14) \qquad \frac{d\theta}{dn} = \frac{\lambda}{cD \cos i} = \frac{\sin i - \sin \theta}{c \cos i}.$$

Since, in equation (13), c is not determinable it is necessary to com-
pare increments $\Delta dn/d\theta$ in terms of the corresponding increments
Δc, whence

$$(15) \qquad \Delta(dn/d\theta) = \left(\cos i/\lambda \tan \frac{i - \theta}{2} \right) \Delta c.$$

Table I. also contains data of this kind computed separately for the
Fraunhofer D, C, etc., employed and their mean values.   To find
the mean width of fringes between these lines, their angular devia-
tions were divided by the number of fringes counted between them
at different values of c.   The results agree as closely as the difficulty
of the observations warrants.   One may note that without remov-

ing N, the corresponding coefficients would be $\Delta d (n + N)/d\theta$, and these are much more in error, here and in the preceding cases. If from $d\theta/dn$, e is eliminated in terms of $(n + N)$ the equation is

$$(16) \qquad \frac{d\theta}{dn} = \frac{\lambda}{D} \frac{1}{(n + N_0) \cos i},$$

so that for a given value of i, θ, N_0, they decrease in size with n. If $n = 0$ they reach the limiting size

$$\frac{d\theta}{dn} = \frac{\lambda}{D N_0 \cos i}.$$

If the crack $N_0 D$ should be made infinitely small, they would be infinitely large. To pass through infinity, N_0 must be negative, which has no meaning for $i > \theta$ or would place one effective edge of the crack S behind the other. These inferences agree with the observations as above detailed. If, however, $i < \theta$, a negative value of N_0 restores equation (16) for $n = 0$ to equation (17), as was actually observed (Figs. 2 and 3).

Finally equation (14) might be used for observation in the incremented form

$$(17) \qquad \Delta(de/d\theta) = \frac{D \cos i}{\lambda} \Delta e ;$$

but I did not succeed with it. One loses track of the run of a fringe with de.

11. *Colored Slit Images and Disc Colors of Coronas.*—In the above experiment the fringes were but a few minutes apart. It is obvious, however, that if N_0 is sufficiently small the fringes will grow with decreasing n, in angular magnitude, until there are but a few black bands in the spectrum. Under these circumstances the undeviated image of the superimposed slits must appear *colored,* particularly so if an effect equivalent to N_0 is present throughout the grating. This phenomenon of colored slits is apparently of interest in its bearing on the theory of coronas, where there is also an interference phenomenon superimposed upon a diffraction phenomenon, as is evidenced by the brilliant disc colors. For instance suppose

a corona were produced by a sufficient number of fog particles distributed throughout a plane normal to the undeviated rays. Now let the alternate particles be moved *in the same way* slightly to the rear of their original position and let the distance between the two planes be small relatively to the wave length of light. In such a case there should be two identical coronas, superimposed in all their parts and they should therefore interfere. Inasmuch, however, as even small fog particles are of the order of size of .0001 cm. and their mean distance apart fifty times larger, *i. e.*, .005 cm., it remains to be proved whether such an effect can be looked to as an explanation of the disc colors of coronas.

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